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A NUMERICAL INVESTIGATION OF THE COMPRESSIVE STRENGTH OF MASONRY USING DISCRETE ELEMENT METHOD

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Abstract. *This paper presents a new detailed micro-modelling approach based on the discrete element method for the analysis of the compressive strength of masonry prisms. According to the method, masonry is represented as an assemblage of Voronoi (in 2D) blocks bonded through zero thickness interface elements. In order to evaluate the ability of the proposed numerical approach for simulating brittle fracture, a series of compressive strength tests is performed and results are compared against those obtained from experimental results found in the literature. The significant advantage of the approach is its ability to model cracking as a real discontinuity between particles of brick and mortar. Also, reliable prediction of small scale experimental tests can reduce the need of costly and timely experimental testing and avoid the reliance on conservative empirical formulas.*

1 INTRODUCTION

Since masonry are usually stressed in compression, the compressive strength of masonry is very important for the design and safety assessment. The code of practice e.g. [1, 2] suggest to estimate the compressive strength of masonry based on the compressive strength of masonry unit and mortar. Although this empirical approach is based on a large set of experimental test results, considering the huge variability in masonry constituents [3], such approach can be highly conservative. For example, existing formulas are not applicable to rubble or irregular masonry which is typical for historical masonry structures. Therefore, the solution today is to carry out a series of small scale experimental tests on masonry prisms and wallets [4]. As experimental research is prohibitively expensive, it is fundamentally important to have available a computational model that can be used to predict the in-service and near-collapse behaviour with sufficient reliability. Once such a model has been established it can be used to investigate a range of complex problems and scenarios that would not, otherwise, be possible.

Over the last few decades, a significant effort has been put on the development of numerical approaches and tools to reproduce the experimental behaviour of the masonry composite under compression [5, 6, 7]. Such models range from considering masonry as anisotropic continuum (macro-models) to the more detailed ones considering masonry as an assemblage of masonry units and mortar (micro-models) [8]. Standard continuum finite element models based on the plasticity and cracking were used by several researchers [9, 10]. However, research carried out by Pina-Henriques and Lourenco [11] demonstrated that results obtained from such numerical models tend to overestimate by approximately 170% the experimental strength and peak strain of masonry prisms tested in compression. In addition, when using such models, the crack initiation and propagation is difficult to be obtained since cracking is simulated in a phenomenological approach. On the other hand, discontinuum modelling approaches which consider the micro-structure of quasi-brittle material such as masonry have been effectively used to study the compressive strength of masonry [13].

Over the last five decades, several advanced models have been developed based on the micro-modelling approach using finite element method and discrete element methods of analysis etc. For an exhaustive discussion about the numerical methods available based on the simplified micro-modelling approach, the reader is requested to refer to [14, 15]. However, such models can only be used when the crack path is known in advance from experiments and interface elements can be used as predetermine crack paths.

Recently, discrete element models [16, 17, 18] have become a very popular tool for the analysis of brittle materials, allowing a suitable representation of its discontinuous nature and marked nonlinear behaviour. Within discrete element method, the domain of interest can be treated as an assemblage of rigid or deformable blocks/particles/bodies. In addition, the DEM is capable of analysis many interacting deformable continuous, discontinuous or fracturing bodies undergoing large displacements and rotations that is widely used in geomechanics. The formulation of the method was proposed initially by Cundall [19] for the study of jointed rock, modelled as an assemblage of rigid blocks. Later this approach was extended to other fields of engineering requiring a detailed study of the contact between blocks or particles such as soil and other granular materials [20]. More recently the approach was applied successfully to model historic masonry structures in which the collapse modes were typically governed by mechanisms in which the deformability of the blocks plays little or no role at all [21, 22].

The aim of this paper is to present the development of a computational model based on the discrete element method which could provide a further insight into the compressive behaviour of quasi-brittle materials such as masonry and allow for a reliable estimation of their compressive strength of and failure pattern.

2 THE PROPOSED METHOD: REPRESENTATION OF MASONRY UNITS AND MORTAR

The proposed model aims to capture the failure mechanism and quasi-brittle behaviour of masonry under compression. It is developed in a discrete element framework using the two dimensional code UDEC (Universal Distinct Element Code) developed by Itasca.

The discontinuous nature of masonry components (e.g. bricks and mortar) is considered here by discretizing bricks and mortar by randomly sized polygonal blocks (i.e. Voronoi elements). The blocks which can be either rigid or deformable can be separated by interfaces which can be viewed as contacts. Such contacts represent in fact grain-interface or grain cementation properties for bricks and mortar accordingly. In this way, the discontinuity can be treated as boundary condition. A force-displacement law is applied at contacts to find the contact force from the known displacements. Newton's law is applied to estimate the motion of the blocks resulting from the known forces acting on then. A representation of masonry units and mortar joints by means of polyhedral elements is presented in the Figure 1 below.

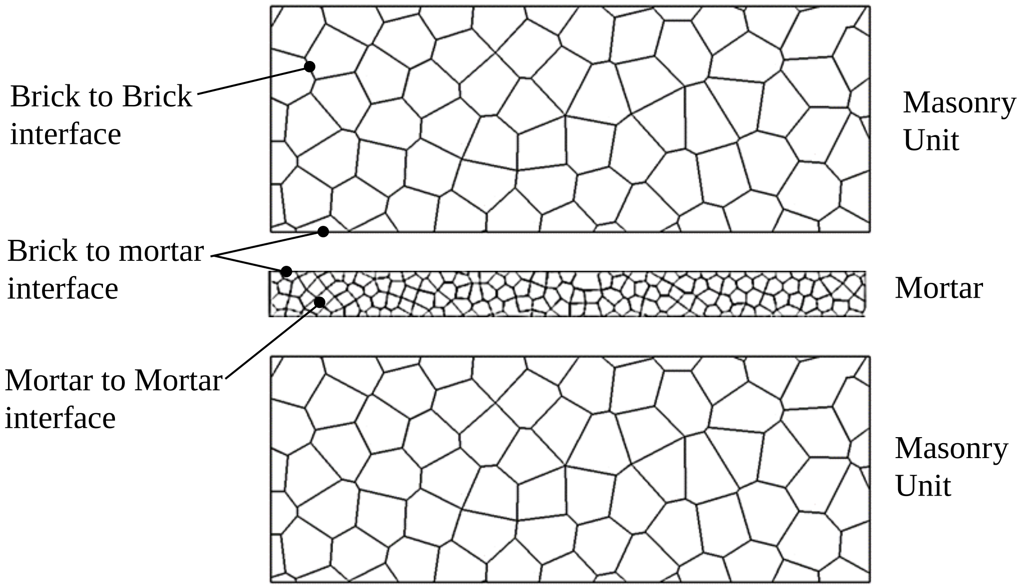


Figure 1: Representation of masonry units and mortar.

From Figure 1, there are three different interfaces between Voronoi elements. These are: a) brick to brick interface; b) mortar to mortar interface; c) brick to mortar interface. Failure can occur at the interfaces (or at the pre-existing fractures) of the Voronoi elements. The smaller the size of the Voronoi elements, the more accurate is the location and propagation of cracking. The mechanical behaviour of Voronoi elements is controlled by Coulomb friction law. In the normal and shear direction, the mechanical behaviour at the zero thickness interfaces is governed by the following two equations:

$$\Delta F_n = - JK_n \cdot \Delta U_n \cdot A_c \tag{1}$$

$$\Delta F_s = - JK_s \cdot \Delta U_s \cdot A_c \tag{2}$$

, where ΔF_n and ΔF_s are the normal and shear force increment; JK_n and JK_s are the normal and shear stiffness; ΔU_n and ΔU_s are the normal and shear displacement accordingly; and Ac is the contact area. In the normal direction, a limiting tensile strength, T, is assumed for the contact; if this value exceeds, then normal stress becomes zero and the Voronoi elements have been detached from each other. In the shear direction, the response is driven by a constant shear stiffness. The shear stress, τ_s, is determined by combining of contact properties, cohesive strength (c) and frictional resistance (φ). This, if:

$$|\tau_s| \leq c + \sigma_n \cdot \tan(\varphi) = \tau_{max} \tag{3}$$

then

$$\Delta\tau_s = - K_s \cdot \Delta U^e_s \tag{4}$$

or else, if

$$|\tau_s| \geq \tau_{max} \tag{5}$$

then

$$\tau_s = \text{sign}(\Delta U^e_s) \cdot \tau_{max} \tag{6}$$

, where ΔU^e_s is the elastic component of the incremental shear displacement and ΔU_s is the total incremental shear displacement. Mechanical properties such as shear stiffness, normal stiffness, cohesive strength, tensile strength and frictional resistance can be assigned to the contacts. Cracks will occur at the contact when the stress applied on the contact exceeds either its' tensile or shear strength. This represents the fracturing of brick or mortar through intact material. In this study, the Coulomb-slip model is employed for the contacts, so that they may fail either in shear or in tension based on the stress inducing failure. The mechanical representation at contacts is shown in Figure 2.

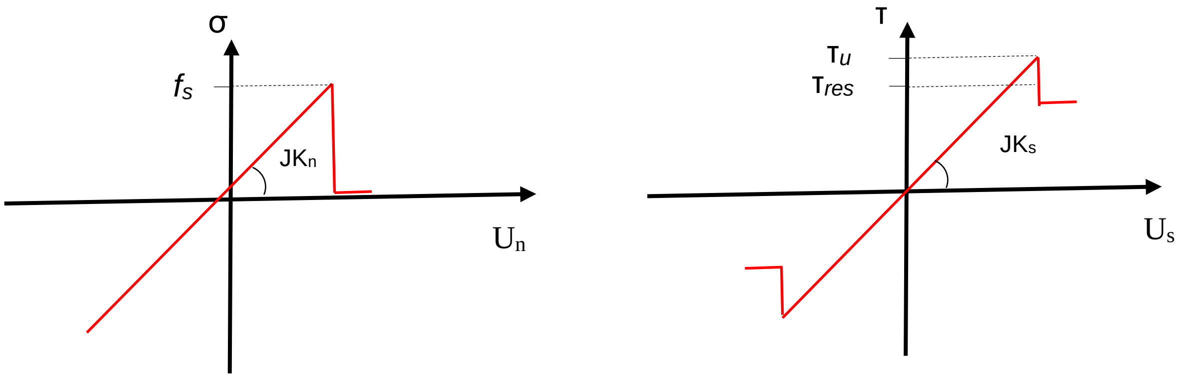


Figure 2: Mechanical representation of contacts at the interfaces.

3 DEVELOPMENT OF THE COMPUTATIONAL MODEL

Since masonry structures are primarily stressed in compression, there has been an extensive concentration of interest to understand their compressive strength. A series of experimental tests have been carried out on standardized masonry prisms subjected to direct compression [3, 6]. Although compressive strength tests on masonry prisms do not necessarily reproduce the state of stress in the actual masonry structure, such tests can be used to obtain values in the selection of design stresses as well as assist in investigating the influence of masonry constituents on the compressive strength. In addition, observations obtained from testing masonry prisms subjected to uniform compression demonstrated that the failure mode of masonry prism depends on the relative strength of mortar compared to the unit. In particular, masonry fails by the development of tension cracks parallel to the axis of loading or due to shear failure along certain lines of weakness.

3.1 Experimental testing

In order to validate the suitability of the computational model, results obtained from experimental tests will be used and compared against those obtained from the literature. In this study the experimental work carried out by Oliveira [6] will be used. Five bricks high bonded with mortar were constructed in the laboratory. The size of the prims were 285 x 130 x250 mm^3. The stacked bond prism were built in accordance to code LUM B1 [23]. Bricks used had all the same dimensions of 285 x 130 x 50 mm^3. Bricks bonded together by mortar. All joints were kept at a uniform thickness and equal to 10 mm.

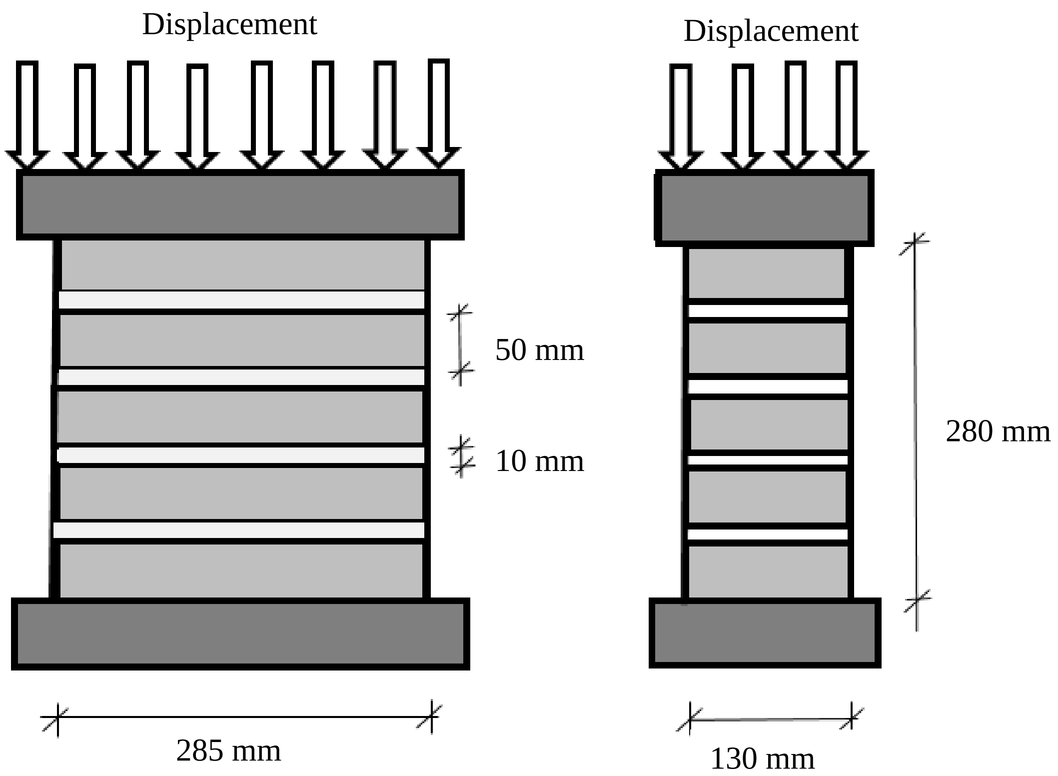


Figure 3 Mechanical representation of contacts at the interfaces.

The brick masonry prims were tested in an axial displacement control machine until failure occurred. Also, three axial LVDTs were placed equally between the machine platens. In this way, both the applied load and displacements at the specimens recorded. The stress strain diagrams of the four masonry prisms are shown in Figure 4. The axial displacement of each prism was defined by the average value computed from the three axial LVDT's used. The axial strain calculated by dividing the average axial displacement by the initial axial length of the prism. The failure modes obtained from the end of the testing are presented in Figure 5.

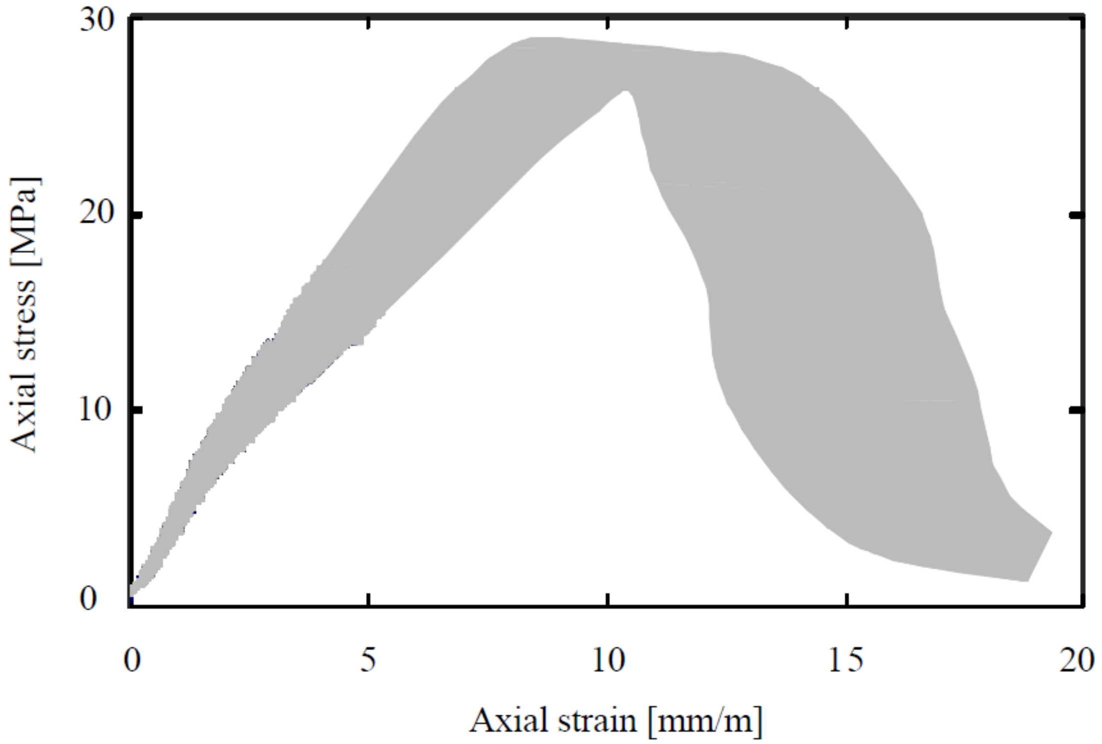


Figure 4: Stress-strain diagram of the four specimens tested in the laboratory.

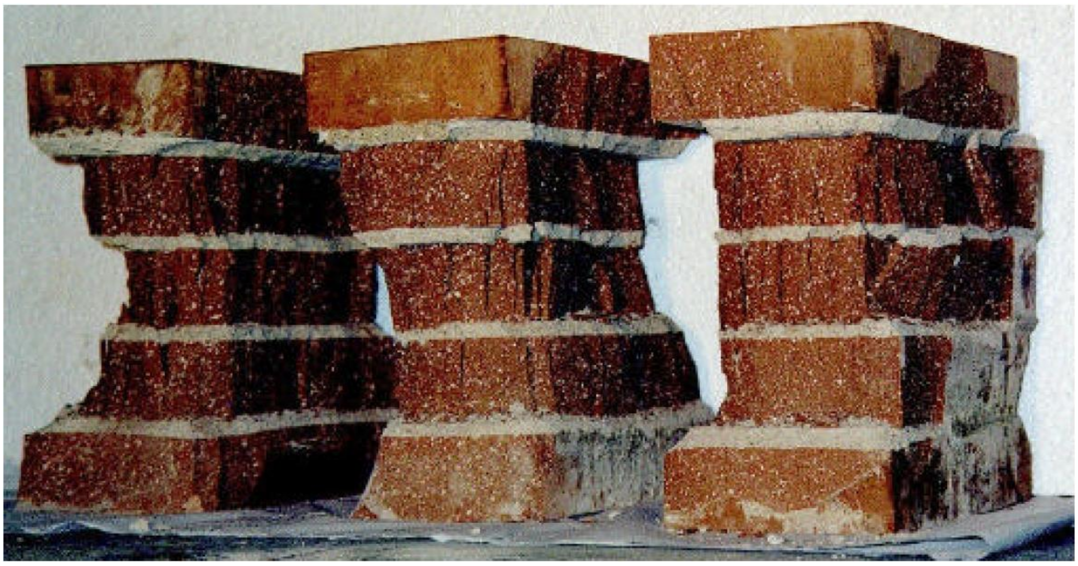


Figure 5: Failure modes of the three tested brick masonry prisms.

3.2 Development of the computational model

Geometric models developed in the computational model to represent the size of the prisms tested in the laboratory (Figure 6). Initially, both bricks and mortar represented by an assemblage of 10 mm and 3 mm size deformable Voronoi elements respectively. Steel platens modeled as rigid (i.e. non-deformable) elements. The material properties used for the development of the computational model are shown in Table 1, 2 and 3. Different material properties have been assigned form the mortar to mortar interface (Table 2) and brick to brick interface (Table 4). The base platen of the prism kept fixed in the vertical and horizontal direction. Gravitational load applied to the system and the model was brought into equilibrium. Then an external velocity equal to 0.01 mm/sec applied at the top face of the top platen. A high damping applied so that the sample remains in a quasi-static condition. During the numerical simulation, histories of axial stress and axial strain recorded.

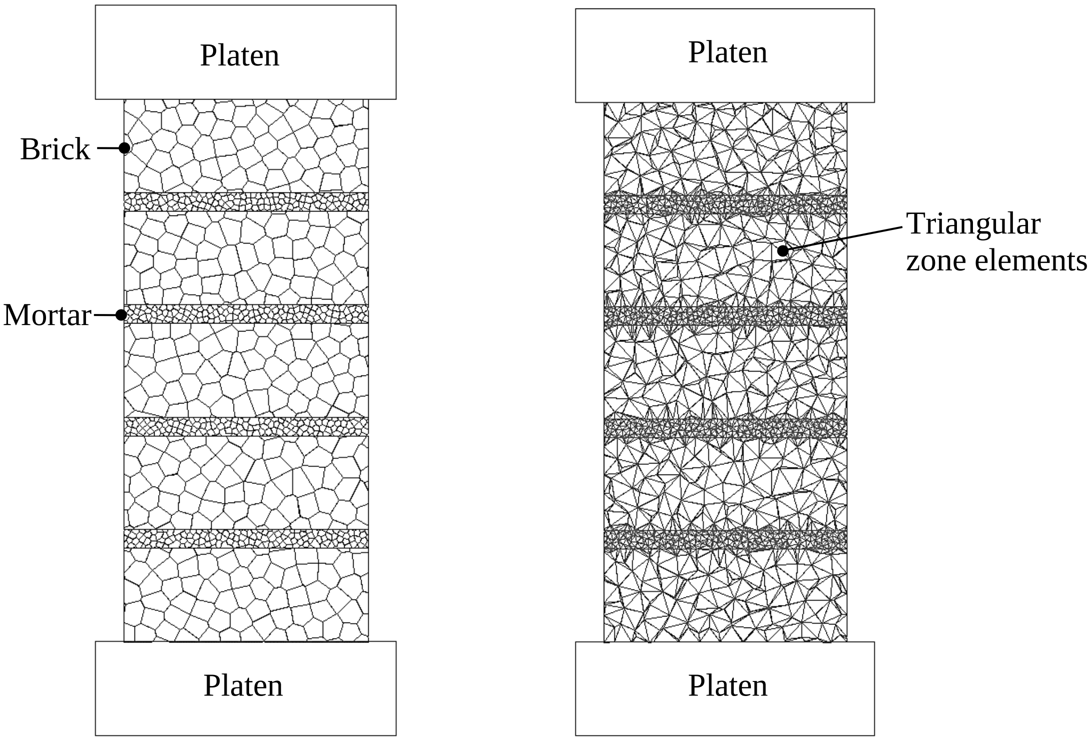


Figure 6: Failure modes of the three tested brick masonry prisms.

Brick properties			**Mortar Properties**		
Young's Modulus (MPa)	Poisson's ratio	Unit Weight (kg/m^3)	Young's Modulus (MPa)	Poisson's ratio	Unit Weight (kg/m^3)
20,000	0.2	2,000	5,800	0.2	2,400

Table 1: Material properties for the brick and mortar.

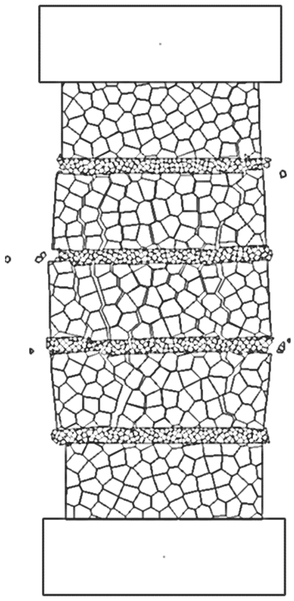
Joint Normal Stiffness (GPa/m)	Joint Shear Stiffness (GPa/m)	Joint frictional resistance (deg.)	Joint cohesive strength (MPa)	Joint cohesive residual strength (MPa)	Joint tensile strength (MPa)
648	374	15	1.4	0.08	0.7

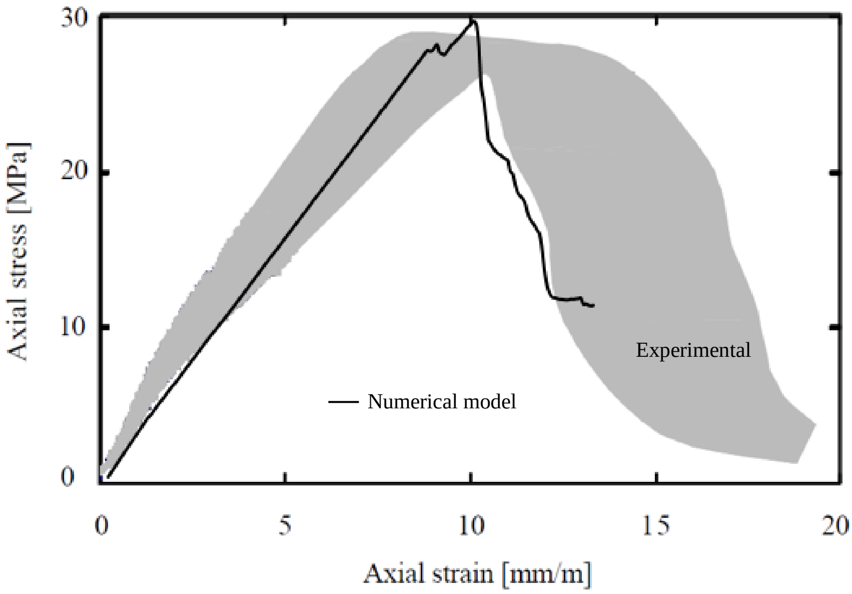
Table 2: Material properties of the mortar to mortar and brick to mortar interface.

Joint Normal Stiffness (GPa/m)	Joint Shear Stiffness (GPa/m)	Joint frictional resistance (deg.)	Joint cohesive strength (MPa)	Joint cohesive residual strength (MPa)	Joint tensile strength (MPa)
1260	630	20	15	3	7.5

Table 3: Material properties of the brick to brick interface.

Figure 7a present the failure mode of the masonry prism and compares the numerical against the experimental axial stress versus axial strain results. From Figure 7a, through the randomly-generated network of potential fractures, failure observed to occur at the brick, mortar and brick-to-mortar interfaces. Vertical cracks appeared in the specimen which is in similar with the experimental failure mode. Also, Figure 7b compares the experimental against the numerical stress-strain curve. From Figure 7b, the stress versus strain relationship of the prism obtained from the numerical model is in a good agreement with the one obtained from the experimental study. In particular, the proposed numerical model is capable to capture both the stiffness and peak stress of the masonry prism.





(a) (b)

Figure 7: Failure modes of the masonry prism (a) as obtained from the numerical model and (b) comparison of experimental against numerical stress vs strain relationship.

In addition, further simulations were carried out with the aim to investigate the influence of the size of the Voronoi elements into the peak strength and failure mode. The size of the Voronoi element for the mortar increased to 7.5 mm while the size of the Voronoi element in the brick kept constant and equal to 10 mm. Figure 8a to 8c presents the evolution of cracking as obtained from the numerical model for the case that the mortar joint is composed with Voronoi elements with size approximately equal to 7.5 mm. Also, Figure 8d compares the stress vs strain relationship for the models using different in size Voronoi elements. From Figure 8d, as the size of the Voronoi increases, the peak strength of the masonry prism increases. Moreover, as the size of Voronoi increases, the stiffness in the masonry prism increases. Finally, for the larger size of Voronoi elements, a ductile behaviour observed close to peak strength which is opposed to a brittle failure when the size of the Voronoi element is smaller.

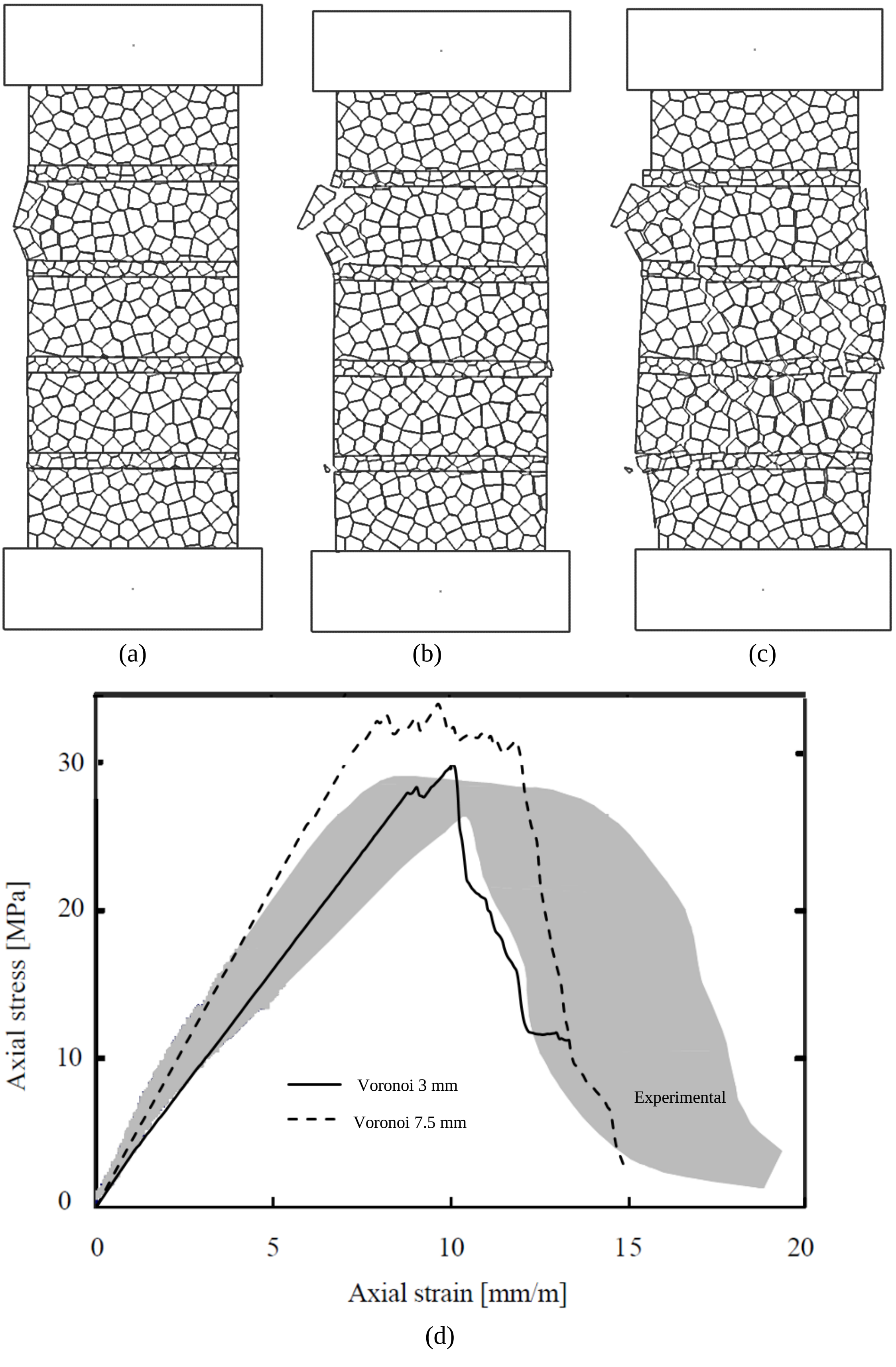


Figure 8: Evolution of failure (a, b, c) in the masonry prism with size of Voronoi element for the mortar equal to 7.5 mm and comparison of experimental against numerical axial stress versus axial strain relationship.

4 CONCLUSIONS

Numerical modelling is nowadays being extensively used for simulating the mechanical behaviour of masonry. Most of the numerical models are focusing either on the macro-modelling approach where masonry is homogenized in the continuum or in the micro-modelling approach where mortar is represented by zero thickness interface while the masonry units are slightly expanded in size in order to keep the geometry unchanged. In both cases, crack propagation is represented using a phenomenological approach smeared in the continuum or following a predefined path (i.e. the mortar joints/zero thickness interface).

A new detailed micro-modelling approach based on the principles of the discrete element method is proposed here. The approach is used to simulate the compressive strength of masonry prisms. According to the method, both masonry units and mortar joints are represented as an assemblage of different in geometry polygonal elements i.e. Voronoi (in 2D) which are bonded together by zero thickness interface elements.

The blocks which can be either rigid or deformable can be separated by interfaces which can be viewed as contacts. Such contacts represent the grain-interface or grain cementation properties for bricks and mortar accordingly. In this way, the discontinuity can be treated as boundary condition.

The numerical method proposed here was validated against experimental results obtained from a five brick masonry prism found in the literature. The experimental and numerical axial stress versus axial strain relationship and the failure mode were compared. Good agreement between the numerical and experimental results obtained. The significant advantage of the proposed approach is its ability to naturally model crack initiation and propagation as a real discontinuity between particles of brick and mortar. Also, reliable prediction of small scale experimental tests can reduce the need of costly and timely experimental testing and avoid the reliance on conservative empirical formulas. In addition, the size of the Voronoi element to represent mortar has been changed and found to influence the failure mode and peak strength. Therefore, the global mechanical characteristics of the masonry prism are significantly influenced by the micro-parameters and size of Voronoi element.

Further studies are planned to develop a calibration methodology for the micro-properties to represent the macro-behavior of the masonry specimen under compression. In addition, studies on more complex experimental tests and dynamic loading condition will be carried out.

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